LIMIT STATES DESIGN IN STRUCTURAL STEEL

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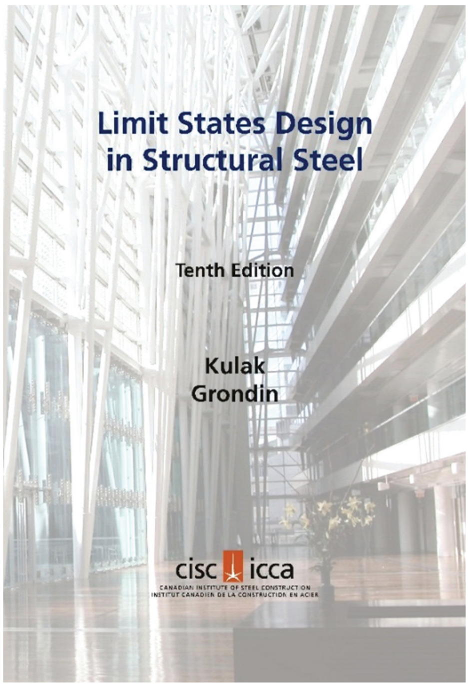
Revisions and updates which will be incorporated into the 10th Edition, 3rd Revised Printing of *Limit States Design in Structural Steel* are highlighted on the following pages. Minor editorial corrections are not shown.



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A first-order analysis of the frame with only gravity load shows that the lateral deflection at the top of the frame is 19.3 mm. With a unit lateral load applied at E, the calculated lateral deflection is 0.894 mm. (This information is used following for the iterative process.)

Iteration 1

Using Equation 8.15,

$$V'^{1}_{i} = \frac{\sum C_i}{h_i}\left(\Delta_{i+1} - \Delta_i\right) = \frac{1100\ \text{kN}}{6000\ \text{mm}}\left(\boxed{55.5\ \text{mm}} - 0\right) = 10.2\ \text{kN}$$

Since the frame consists of only one storey, the fictitious horizontal force obtained using Equation 8.16 is equal to the fictitious shear, namely, 10.2 kN. This fictitious lateral load is added to the applied lateral load, including the notional load of 5.5 kN. The new lateral load is therefore 50.7 kN. Since the deflection is obtained from a first-order analysis, the results of the first-order analyses presented above can be used to determine the lateral deflection at E as follows:

$$\Delta^{1}_{top} = 19.3\ \text{mm} + \left(0.894\ \text{mm} / \text{kN} \times \boxed{50.7\ \text{kN}\right) = 64.6\ \text{mm}}$$

Iteration 2

Using the deflection calculated in the first iteration, the fictitious shear and lateral load are updated.

$$V'^{\,2}_{i} = \frac{1100\ \text{kN}}{6000\ \text{mm}}\boxed{\left(64.6\ \text{mm}\right.} \left. - 0\right) = 11.8\ \text{kN}$$

$$H'^{\,2}_{i} = 11.8\ \text{kN}$$

The total lateral load is 40.5 kN + 11.8 kN = 52.3 kN

$$\Delta^{2}_{top} = 19.3\ \text{mm} + \left(0.894\ \text{mm} / \text{kN} \times 52.3\ \text{kN}\right) = 66.1\ \text{mm}$$

Iteration 3

Using the deflection calculated in the first iteration, the fictitious shear and lateral load are updated.

$$V'^{\,3}_{i} = \frac{1100\ \text{kN}}{6000\ \text{mm}}\left(66.1\ \text{mm} - 0\right) = 12.1\ \text{kN}$$

$$H'^{\,3}_{i} = 12.1\ \text{kN}$$

The total lateral load is 40.5 kN + 12.1 kN = 52.6 kN

$$\Delta^{3}_{top} = 19.3\ \text{mm} + \left(0.894\ \text{mm} / \text{kN} \times 52.6\ \text{kN}\right) = 66.3\ \text{mm}$$

This deflection is within <1.0 % of the deflection calculated in the second iteration, and the analysis has converged. The second-order effects are now obtained from a first-order analysis with a lateral load of 52.6 kN applied as shown in Figure 8.12(b). The results agree well with those obtained using the amplification factor method. It should be noted that some of the reaction forces are not in complete agreement because the reaction forces shown in Figure 8.12(b) were determined from a first order analysis with the amplified lateral load, i.e., equilibrium was considered on the undeformed structure rather than on the deformed structure.

(c) Negative Brace Area Method

The negative brace area is obtained from Equation 8.17.

$$A_o = -\frac{\sum C_f}{h}\frac{L_o}{E\cos^2\alpha}$$

where L_o is equal to 13.42 m, the length of the diagonal from B to D.

$$A_o = -\frac{1100\times10^3\ \text{N}}{6000\ \text{mm}}\times\frac{13\,420\ \text{mm}}{200\,000\ \text{MPa}\left(\dfrac{12\ \text{m}}{13.42\ \text{m}}\right)^2} = -15.4\ \text{mm}^2$$

A first-order analysis of the modified frame with the factored and notional loads leads to the same results as those presented in Figure 8.12(b).

(d) Finite Element Method

Several commercially available structural analysis software will perform a P–Δ analysis. Although different software developers have implemented the stiffness method slightly differently for P–Δ analysis, the results are similar for most practical problems. The P–Δ analysis is performed with the factored loads and the notional load applied simultaneously on the structure. The results of an analysis conducted using the commercial software S-FRAME are presented in Figure 8.13.

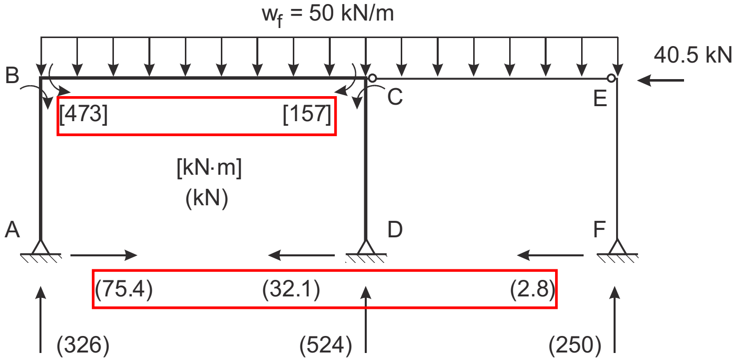


Figure 8.13 – Results of Second-Order Analysis — Finite Element Method

A comparison of the results obtained using different methods of P–Δ analysis indicates that all approximate methods lead to approximately the same end results. Obviously, the use of commercial software to perform an exact elastic second-order analysis is the least time consuming. However, when second-order analysis software

$H_{Ht} = 15.0\,\text{kN} + 6.9\,\text{kN} + 0.0 = 21.9\,\text{kN}$

The results of a first-order analysis for the lateral load case are presented in Figure 8.15(b). The corresponding first storey sway is 22.3 mm and the second storey sway, that is, the difference between the deflections at the roof level and second floor level, is 12.2 mm.

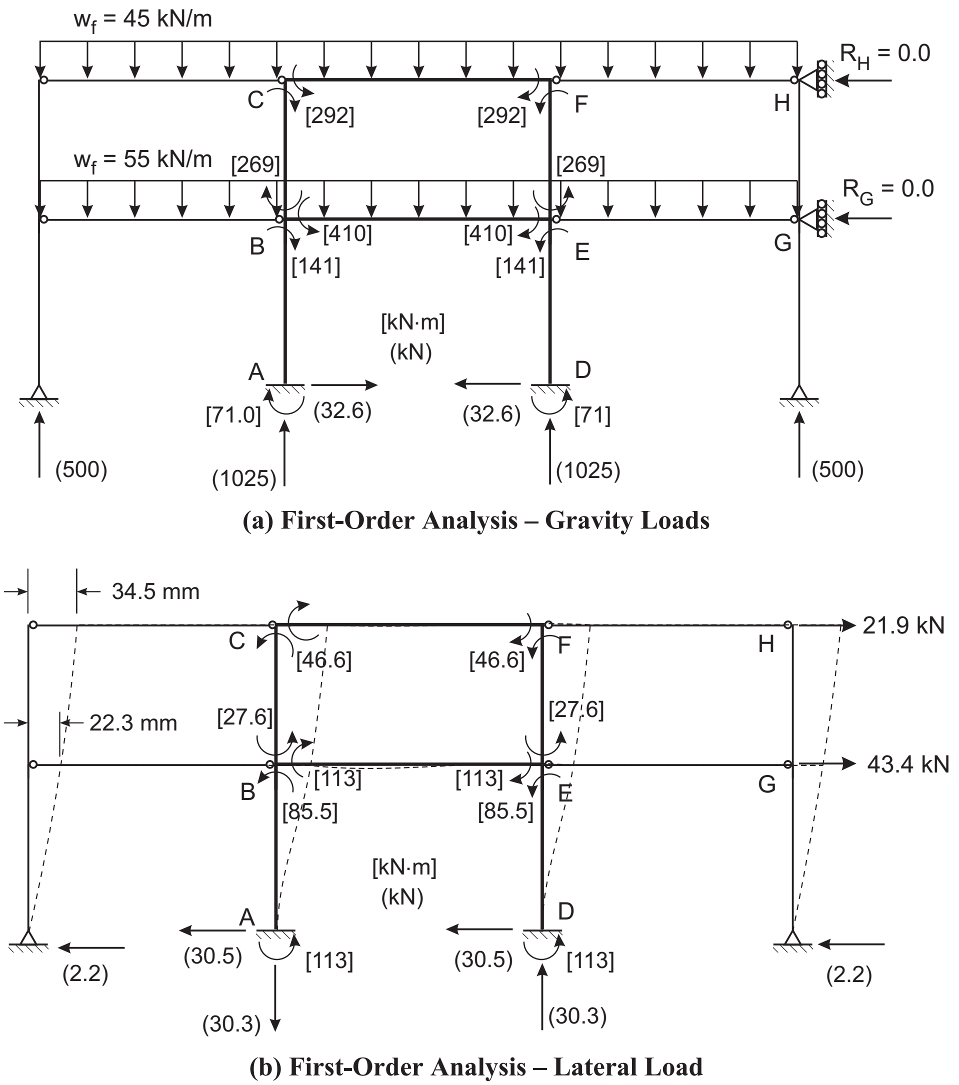
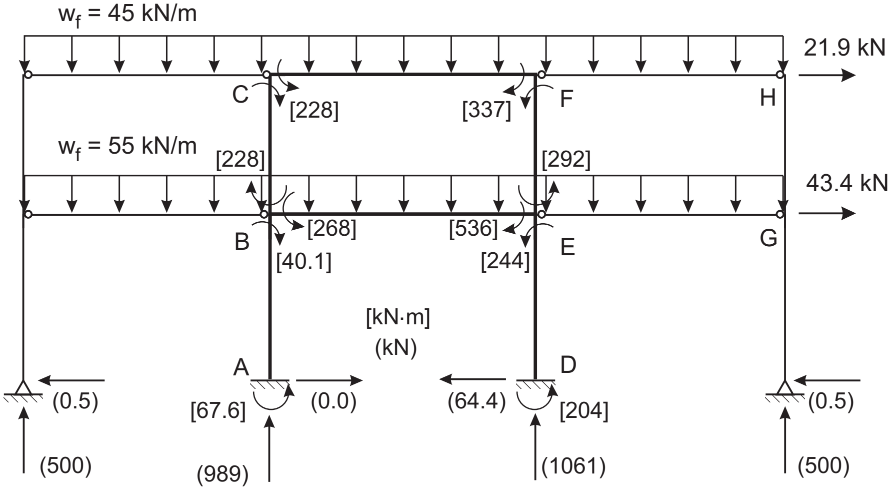


Figure 8.15 – Analysis Results for Frame of Example 8.2 — Amplification Factor Method



(c) Second-Order Analysis Results – Amplification Factor Method

Figure 8.15 – (cont'd)

(a) Amplification Factor Method

The amplification factor, U_2, is calculated for each storey using

$$U_2 = \frac{1}{1 - \dfrac{\sum C_f \; \Delta_f}{\sum V_f \; h}}$$

where, for the first storey

$$\sum C_f = (55\,\text{kN/m} + 45\,\text{kN/m}) \times (10\,\text{m} + 10.5\,\text{m} + 10\,\text{m}) = 3050\,\text{kN}$$

This value can also be obtained by adding the axial force in all the columns of the first storey.

$\dfrac{\Delta_f}{\sum V_f}$ is the flexibility of the storey, obtained from the results of the first-order analysis.

$$\frac{\Delta_f}{\sum V_f} = \frac{\boxed{22.3\,\text{mm}}}{21.9\,\text{kN} + 43.4\,\text{kN}} = \boxed{0.342\,\text{mm / kN}}$$

h is the storey height (= 6500 mm)

$$U_2 = \frac{1}{1 - \dfrac{\sum C_f \; \Delta_f}{\sum V_f \; h}} = \frac{1}{1 - \dfrac{3050\,\text{kN}}{6500\,\text{mm}} \boxed{0.342\,\text{mm / kN}}} = \boxed{1.191}$$

Similarly, for the second storey,

$$\sum C_f = (45\,\text{kN / m}) \times (10\,\text{m} + 10.5\,\text{m} + 10\,\text{m}) = 1372\,\text{kN}$$

$$\frac{\Delta_f}{\sum V_f} = \frac{\boxed{(34.5 - 22.3)\text{ mm}}}{21.9\text{ kN}} = \boxed{0.557\text{ mm/kN}}$$

For a storey height of 5500 mm,

$$U_2 = \frac{1}{1 - \dfrac{\sum C_f\ \Delta_f}{\sum V_f\ h}} = \frac{1}{1 - \dfrac{1372\text{ kN}}{5500\text{ mm}} \times \boxed{0.557\text{ mm/kN}}} = \boxed{1.162}$$

The second-order moments for the beam-columns located in the first storey can be obtained from Equation 8.13. When adding moments, a clockwise moment is taken as a positive moment.

$$\boxed{\begin{aligned}
M_f^{AB} &= M_{fg}^{AB} + U_2\ M_{ft}^{AB} = 71\text{ kN}\cdot\text{m} + 1.191 \times (-113\text{ kN}\cdot\text{m}) = -63.6\text{ kN}\cdot\text{m} \\
M_f^{BA} &= M_{fg}^{BA} + U_2\ M_{ft}^{BA} = 141\text{ kN}\cdot\text{m} + 1.191 \times (-85.5\text{ kN}\cdot\text{m}) = 39.2\text{ kN}\cdot\text{m} \\
M_f^{DE} &= M_{fg}^{DE} + U_2\ M_{ft}^{DE} = -71\text{ kN}\cdot\text{m} + 1.191 \times (-113\text{ kN}\cdot\text{m}) = -206\text{ kN}\cdot\text{m} \\
M_f^{ED} &= M_{fg}^{ED} + U_2\ M_{ft}^{ED} = -141\text{ kN}\cdot\text{m} + 1.191 \times (-85.5\text{ kN}\cdot\text{m}) = -243\text{ kN}\cdot\text{m}
\end{aligned}}$$

Similarly, for the columns in the second storey,

$$\boxed{\begin{aligned}
M_f^{BC} &= M_{fg}^{BC} + U_2\ M_{ft}^{BC} = 269\text{ kN}\cdot\text{m} + 1.162 \times (-27.6\text{ kN}\cdot\text{m}) = 237\text{ kN}\cdot\text{m} \\
M_f^{CB} &= M_{fg}^{CB} + U_2\ M_{ft}^{CB} = 292\text{ kN}\cdot\text{m} + 1.162 \times (-46.6\text{ kN}\cdot\text{m}) = 238\text{ kN}\cdot\text{m} \\
M_f^{EF} &= M_{fg}^{EF} + U_2\ M_{ft}^{EF} = -269\text{ kN}\cdot\text{m} + 1.162 \times (-27.6\text{ kN}\cdot\text{m}) = -301\text{ kN}\cdot\text{m} \\
M_f^{FE} &= M_{fg}^{FE} + U_2\ M_{ft}^{FE} = -292\text{ kN}\cdot\text{m} + 1.162 \times (-46.6\text{ kN}\cdot\text{m}) = -346\text{ kN}\cdot\text{m}
\end{aligned}}$$

A summary of the second-order analysis results using the amplification factor method is presented in Figure 8.15(c). The lateral loads are shown as the applied load plus the notional loads. The reaction forces are determined from equilibrium consideration on the deformed structure.

(b) Fictitious Horizontal Loads Method

A first-order analysis of the structure with the factored loads, including the notional loads, is performed to obtain the storey drift. The results of this analysis are presented in Figure 8.16(a). Fictitious column shears and horizontal forces are calculated using Equations 8.15 and 8.16, respectively.

A first-order analysis of the frame with only lateral loads shows that the lateral deflection at level 2 is 22.3 mm and 34.5 mm at the first floor and the top of the frame.

Iteration 1

Using Equation 8.15,

$$V'_1 = \frac{\sum C_1}{h_1}\left(\Delta_{1+1} - \Delta_1\right) = \frac{3050\ \text{kN}}{6500\ \text{mm}}\boxed{\left(22.3\ \text{mm} - 0\right) = 10.5\ \text{kN}}$$

$$V'_2 = \frac{\sum C2}{h2}\left(\Delta_{2+1} - \Delta_2\right) = \frac{1372\ \text{kN}}{5500\ \text{mm}}\boxed{\left(34.5\ \text{mm} - 22.3\ \text{mm}\right) = 3.04\ \text{kN}}$$

The fictitious horizontal forces at levels 2 and 3 are obtained using Equation 8.16. At level 2, the fictitious lateral load is equal to the difference between the shear force in the first storey and the shear force in the second storey, namely,

$$H'_2 = V'_1 - V'_2 = \boxed{10.5 - 3.04 = 7.46\ \text{kN}}$$

$$H'_3 = V'_2 - V'_3 = \boxed{3.04 - 0 = 3.04\ \text{kN}}$$

These fictitious lateral loads are now added to the applied lateral loads, including the notional loads. The new lateral loads are therefore 50.9 kN and 24.9 kN for levels 2 and 3, respectively. Since the deflections are obtained from a first-order analysis, the principle of superposition can be used to obtain the resulting deflections. Alternatively, the structure can be re-analyzed with the incremented lateral loads. The resulting lateral deflections are:

$$\boxed{\Delta_2 = 25.9\ \text{mm}; \qquad \Delta_3 = 39.9\ \text{mm}}$$

Iteration 2

Using the deflections calculated in the first iteration, the fictitious shears and lateral loads are updated.

$$V'_1 = \frac{3050\ \text{kN}}{6500\ \text{mm}}\boxed{\left(25.9\ \text{mm} - 0\right) = 12.2\ \text{kN}}$$

$$V'_2 = \frac{1372\ \text{kN}}{5500\ \text{mm}}\boxed{\left(39.9\ \text{mm} - 25.9\ \text{mm}\right) = 3.5\ \text{kN}}$$

$$\boxed{H'_2 = 8.7\ \text{kN}; \qquad H'_3 = 3.5\ \text{kN}}$$

The total lateral load at level 2 is 43.4 kN + 8.7 kN = 52.1 kN and the total load at level 3 is 21.9 kN + 3.5 kN = 25.4 kN. The resulting deflections are:

$$\boxed{\Delta_2 = 26.5\ \text{mm}; \qquad \Delta_3 = 40.8\ \text{mm}}$$

Iteration 3

Using the deflection calculated in the first iteration, the fictitious shear and lateral load are updated.

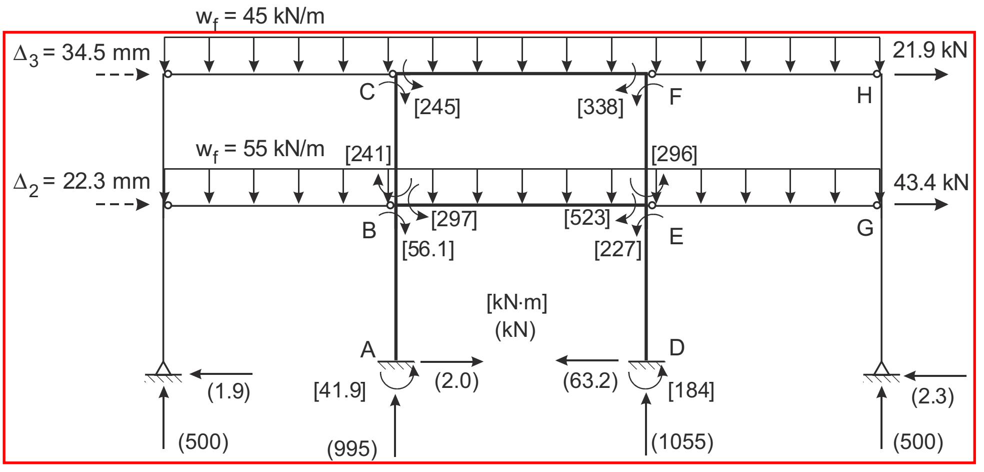
$$V'_1 = \frac{3050\ \text{kN}}{6500\ \text{mm}}\boxed{\left(26.5\ \text{mm} - 0\right) = 12.4\ \text{kN}}$$

$$V'_2 = \frac{1372\ \text{kN}}{5500\ \text{mm}}\boxed{\left(40.8\ \text{mm} - 26.5\ \text{mm}\right) = 3.6\ \text{kN}}$$

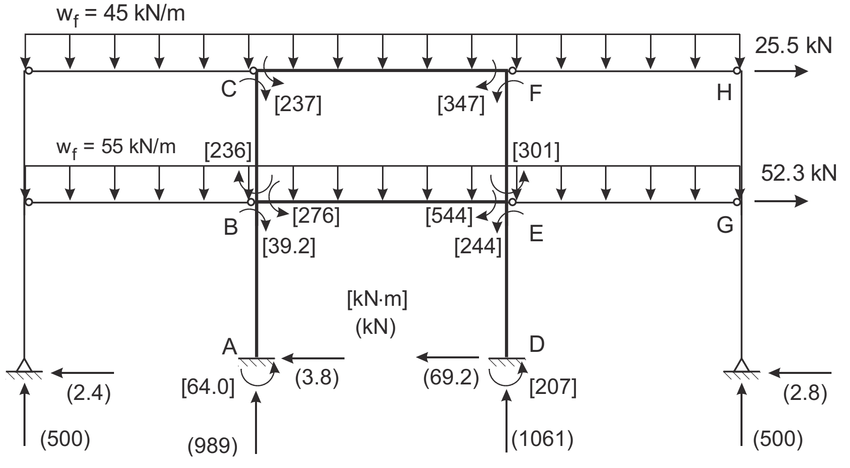
$$H'_2 = 8.8\text{ kN}; \qquad H'_3 = 3.6\text{ kN}$$

The total lateral loads for this iteration are 52.2 kN and 25.5 kN for levels 2 and 3, respectively. The resulting lateral deflections are:

$$\Delta_2 = 26.6\text{ mm}; \qquad \Delta_3 = 40.9\text{ mm}$$



(a) Results of First-Order Analysis – Factored + Notional Loads



(b) Final Results of Iterative Analysis

Figure 8.16 – Analysis Results for Frame of Example 8.2 — Fictitious Lateral Load Method

The deflections are within 1.0 % of the deflections calculated in the second iteration. The problem has therefore converged. The second-order effects can now be obtained from a first-order analysis with the lateral loads calculated in iteration 3. The results, shown in Figure 8.16(b), are in very close agreement with those obtained from the amplification factor method. The reader is reminded that the reaction forces shown in Figure 8.16(b) were determined considering equilibrium on the undeformed structure rather than on the deformed structure.

(c) Negative Brace Area Method

A brace member with a negative area is inserted in the structure in each storey. The area of each brace is obtained from Equation 8.17.

$$A_o = -\frac{\sum C_f}{h} \frac{L_o}{E \cos^2 \alpha}$$

where L_o is equal to 13.35 m for the first storey and 11.85 m for the second storey.

For the first storey, the area of the fictitious bracing member is equal to:

$$A_o = -\frac{3050 \times 10^3 \text{ N}}{6500 \text{ mm}} \times \frac{12\,350 \text{ mm}}{200\,000 \text{ MPa}\left(\frac{10.5 \text{ m}}{12.35 \text{ m}}\right)^2} = -40.08 \text{ mm}^2$$

The area of the fictitious bracing member for the second storey is equal to:

$$A_o = -\frac{1372 \times 10^3 \text{ N}}{5500 \text{ mm}} \times \frac{11\,850 \text{ mm}}{200\,000 \text{ MPa}\left(\frac{10.5 \text{ m}}{11.85 \text{ m}}\right)^2} = -18.83 \text{ mm}^2$$

A first-order analysis of the modified frame with the factored loads and the notional load leads to the same results as those presented in Figure 8.16(b).

(d) Finite Element Method

A P–Δ analysis for the frame of Figure 8.14 was performed with the factored loads and the notional load applied simultaneously on the structure. The results of this analysis, conducted using commercial software, are presented in Figure 8.17.

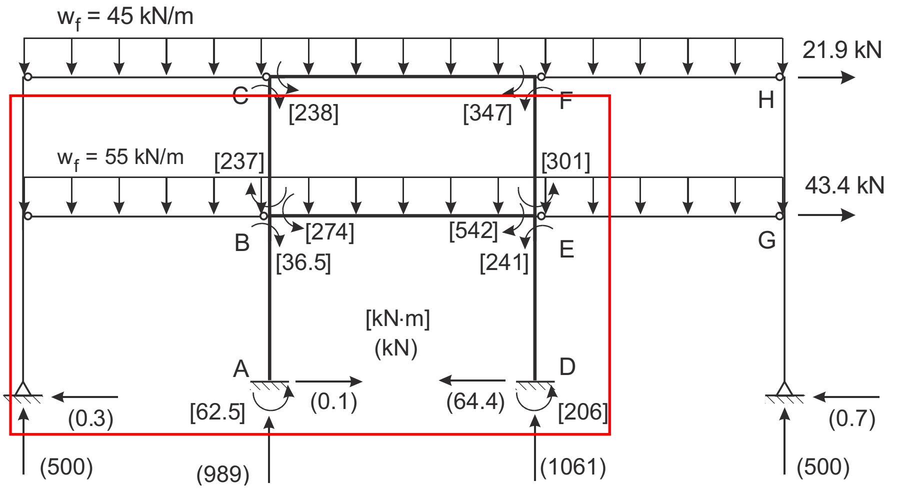


Figure 8.17 – Results of Second-Order Analysis — Finite Element Method