

**Limit States Design in Structural Steel**  
Eighth Edition

**Errata**

Since the cable will pass over saddles and then down to anchorages at the edge of the roof or to the ground, rope will be preferred to strand because of its flexibility. From ASTM A603-98, the breaking (ultimate) strength of a 14.29 mm diameter rope with a so-called Class A galvanized coating is 129 kN. Its factored resistance can be taken as  $0.85 \times 0.9 \times 129 \text{ kN} = 98.7 \text{ kN}$ . The metallic area of the rope is  $94.8 \text{ mm}^2$  and it has a mass of 0.79 kg/m. If prestretched, it will have a minimum modulus of elasticity of  $140 \times 10^3 \text{ MPa}$ .

The resulting shear stress in the flange is maximum at the flange-to-web junction and is obtained as

$$\tau_{bx \text{ flange}} = \frac{V Q}{I t} = \frac{45\,000 \text{ N} \times 448 \times 10^3 \text{ mm}^3}{448 \times 10^6 \text{ mm}^4 \times 20.6 \text{ mm}} = 2.2 \text{ MPa}$$

The corresponding shear stress in the web, which is maximum at the centroid of the cross-section, is given as

$$\tau_{bx \text{ web}} = \frac{V Q}{I t} = \frac{45\,000 \text{ N} \times 1.189 \times 10^6 \text{ mm}^3}{448 \times 10^6 \text{ mm}^4 \times 12.6 \text{ mm}} = 9.5 \text{ MPa}$$

For bending about the weak axis, the shear force over half span and the resulting maximum shear stress in the flanges are given as:

$$V = 12 \text{ kN} / 2 = 6 \text{ kN}$$

$$\tau_{by} = \frac{V Q}{I t} = \frac{6000 \text{ N} \times 96.5 \times 10^3 \text{ mm}^3}{25.1 \times 10^6 \text{ mm}^4 \times 20.6 \text{ mm}} = 1.1 \text{ MPa}$$

The resulting shear stress in the web is negligible.

Warping shear stresses are obtained from the equivalent shear forces in the flanges as shown in Figure 5.17. The shear force over the half span is given as:

$$V = 10.3 \text{ kN} / 2 = 5.2 \text{ kN}$$

The resulting maximum shear stress in the flanges caused by this shear force is given as:

$$\tau_w = \frac{V Q}{I t} = \frac{5200 \text{ N} \times 96.5 \times 10^3 \text{ mm}^3}{12.5 \times 10^6 \text{ mm}^4 \times 20.6 \text{ mm}} = 2.0 \text{ MPa}$$

The pure torsion stresses can be determined using Equation 5.30 and the design charts provided in Reference [5.24]. Figure 5.20 shows the chart applicable to a beam simply supported at both ends and a concentrated torque at midspan. The function  $(d\phi/dx)(GJ/T)$  is plotted as a function of the distance along the span. The variable  $T$  in the torsion function is the applied torque at midspan ( $= 12 \text{ kN} \times 0.3845 \text{ m} = 4.61 \text{ kN} \cdot \text{m}$ ). The dark solid lines represent the torsion function for values of  $\mu L$  varying from 0.5 to 6.0. For the beam considered here,

$$\mu L = \sqrt{\frac{G J}{E C_w}} L = \sqrt{\frac{77\,000 \times 1460 \times 10^3}{200\,000 \times 1260 \times 10^9}} \times 7400 = 4.94$$

The dotted line in Figure 5.20 represents the torsion function for  $\mu L = 4.94$ . The torsion function is maximum at the end supports and zero at midspan. From Figure 5.20 the maximum value of the torsion function is 0.42, from which:

$$\frac{d\phi}{dx} = 0.42 \frac{T}{G J}$$

Substituting into Equation 5.30, we obtain

provide also a powerful tool for the design of steel members. Use of any design software never replaces the need for a solid understanding of the behaviour of the members and the design rules, however.

### 5.15 Beams in Plastically Designed Structures

In a structure designed to resist moments and forces calculated on the basis of a plastic analysis, the beams are required to deliver a moment capacity equal to  $M_p$ . In addition, however, portions of the beams must be able to act as plastic hinges, that is, they must be able to resist a moment of  $M_p$  while at the same time undergoing considerable inelastic rotation. The required behaviour of a plastic hinge region is illustrated by the curve for Class 1 sections in Figure 5.6.

The additional inelastic rotation requirement means that the flange plate will be subjected to larger average strains than will the flange of a Class 2 section. The limiting flange and web slenderness ratios are correspondingly reduced to those appropriate for a Class 1 section. As given in Equations 5.8 and 5.14, the respective limits are:

$$\frac{b_o}{t} \leq \frac{145}{\sqrt{F_y}} \quad \text{and} \quad \frac{h}{w} \leq \frac{1100}{\sqrt{F_y}}$$

Members in plastically designed structures must also be braced laterally so that the full  $M_p$  value can be delivered by the beam and also to ensure that the member can deform inelastically (at plastic hinge locations), as shown by curve B of Figure 5.9.

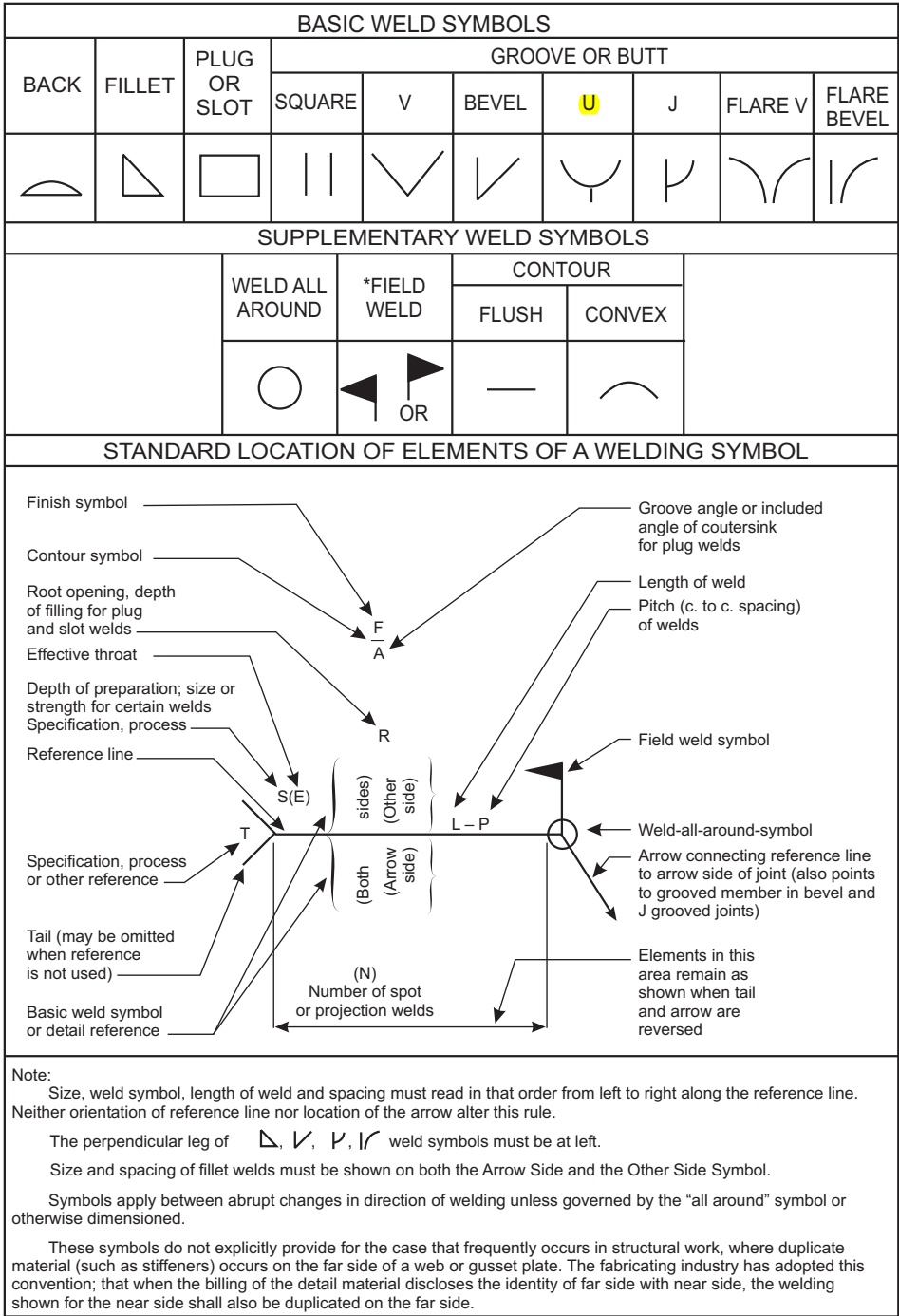
There is a distinct difference between the behaviour of a beam subjected to a uniform bending moment (Figure 5.9) and one subjected to a moment gradient (Figure 5.6). In the uniform moment case, yielding extends over a considerable length of the member as the beam moment approaches  $M_p$  and this weakens the beam significantly with respect to lateral-torsional buckling [5.4]. To achieve the desired behaviour, the distance from a plastic hinge (which must be braced laterally) to the adjacent braced point,  $L_{cr}$ , is limited in Clause 13.7 of S16-01 to:

$$L_{cr} = \frac{r_y (25\,000 + 15\,000\kappa)}{F_y} \tag{5.34}$$

In regions removed from potential plastic hinge locations, these provisions do not apply. In fact, the bracing spacing would be that specified for the same member in a structure designed on the basis of an elastic analysis [5.4].

## References

- 8.1 WRC-ASCE Joint Committee, "Plastic Design in Steel, A Guide and Commentary," 2nd Edition, American Society of Civil Engineers, New York, 1971.
- 8.2 Galambos, T.V., "Structural Members and Frames," Prentice-Hall, Inc. Englewood Cliffs, N.J., 1968.
- 8.3 Austin, W.J., "Strength and Design of Metal Beam-Columns," Journal of the Structural Division, American Society of Civil Engineering, Vol. 87, ST4, April 1961.
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- 8.5 Nixon, D., Beaulieu, D. and Adams, P.F., "Simplified Second Order Frame Analysis," Canadian Journal of Civil Engineering, Vol. 2, No. 4, 1975.
- 8.6 Cook, R. D., Malkus, D. S., and Plesha, M. E., "Concepts and Applications of Finite Element Analysis," 3rd edition, John Wiley & Sons, New York, 1989.
- 8.7 Clarke, M. and Bridge, R., "The Design of Steel Frames Using the Notional Load Approach," Proceedings of the 5th Colloquium on Stability of Metal Structures, Structural Stability Research Council, 1996.
- 8.8 Canadian Standards Association, CAN/CSA-S16-01, "Limit States Design of Steel Structures," Toronto, Ontario, 2001.
- 8.9 Dawe, J.L. and Kulak, G.L. "Local Buckling Behavior of Beam-Columns," Journal of the Structural Division, American Society of Civil Engineers, Vol. 112, ST11, November 1986.
- 8.10 Galambos, T.V. and Ketter, R.L., "Columns Under Combined Bending and Thrust," Journal of the Engineering Mechanics Division, American Society of Civil Engineers, Vol. 84, EM2, April 1959.
- 8.11 Essa, H.E. and Kennedy, D.J.L., "Proposed Provisions for the Design of Steel Beam-Columns in S16-2001," Canadian Journal of Civil Engineering, Vol. 27, No. 4, August 2000.
- 8.12 Canadian Institute of Steel Construction, "Handbook of Steel Construction," Ninth Edition, Toronto, Ontario, 2006.
- 8.13 Kennedy, D.J.L., Picard, A., and Beaulieu, D., "New Canadian Provisions for the Design of Steel Beam-Columns," Canadian Journal of Civil Engineering, Vol. 17, No.6, Dec. 1990.



**Figure 9.6 – Standard Weld Symbols**

The minimum end distance (in the line of stress), i.e., the distance from the last bolt to the end of the connected part, is set by the requirements of Clause 22.3.4 of S16. Where there are more than two bolts in line, this simply refers to Table 6 of the Standard. Assuming the plate end to have been sheared, this minimum end distance for a 22 mm diameter bolt is given as 38 mm.

As explicitly required by the Standard, it is also necessary to check the ultimate capacity of any connection designed as slip-critical. See the concluding statement in part (c) of this example with respect to this check.

*Solution (c)*

For the bearing-type connection and again considering 22 mm diameter A325M bolts, for one bolt the shear resistance (Equation 9.6) is:

$$V_r = 0.60 \times 0.80 \times 1 \text{ shear plane} \times 380 \text{ mm}^2 \times 830 \text{ MPa} \\ = 151 \times 10^3 \text{ N} = 151 \text{ kN}$$

This calculation assumes that the threads are not intercepted by a shear plane, and this must now be checked. The necessary information regarding bolt and nut dimensions can be found in the Handbook (p. 6–158, 159). Helpful tables for the design of bolted connections are also available in the Handbook.

The material that must be accommodated within the bolt grip is a total of  $20 + 20 = 40 \text{ mm}$ . A 22 mm dia. bolt that is 70 mm long (underside of bolt head to end of bolt) will be a suitable choice. Since the thread length on a 22 mm dia. bolt is 38 mm (for bolts  $\leq 100 \text{ mm}$  long), this means that the threads start  $70 - 38 = 32 \text{ mm}$  from the underside of the bolt head. The shear plane is 20 mm from the underside of the head, and therefore the threads are not intercepted by the shear plane. The bolt shear capacity calculated above (151 kN) does not have to be adjusted.

The plate capacity in bearing for one bolt is (Equation 9.5):

$$B_r = 3 \times 0.67 \times 20 \text{ mm} \times 22 \text{ mm} \times 450 \text{ MPa} = 398 \times 10^3 \text{ N} = 497 \text{ kN}$$

(If plates of different thickness had been used, the thickness of the thinner plate would be used in this calculation. If a double shear arrangement is present, then the bolt bears against two plate thicknesses in one direction and one thickness in the other. The combination giving the least thickness is used in calculating the bearing resistance.)

The capacity is governed by the resistance in shear. The number of bolts can now be calculated and, once the joint length is known, the need for reduction in bolt shear capacity due to joint length examined.

$$\text{No. req'd.} = \frac{640 \text{ kN}}{151 \text{ kN/bolt}} = 4.2$$

Use six M22 A325M bolts in two lines, as shown in Figure 9.15(b).

Now that the bolt layout has been established, the bolt shear strength reduction with length can be checked. As noted in Section 9.5, a reduction is required only when  $L > 15 d$ . In this example,  $L = 140 \text{ mm}$  (see Figure 9.15(b)), which is less than  $15 d = 15 \times 22 = 330 \text{ mm}$ , and no reduction is required.

$$d_c w \frac{F_y}{\sqrt{3}} = \frac{M}{d_b}$$

and solving for the web thickness,  $w = \frac{\sqrt{3} M}{d_c d_b f_y}$

Applying the resistance factor, and writing as a requirement, this becomes

$$w \geq \frac{\sqrt{3} M}{\phi d_c d_b f_y} = \frac{1.9 M}{d_b d_c F_y} \quad (9.14)$$

If the web of the beam supplied is not at least equal to the requirement given by Equation 9.14, doubler plates or a diagonal stiffener can be provided. The selection of a new section size may also be economical. The diagonal stiffener, as shown in Figure 9.21(c), is the usual choice.

The stiffener is proportioned by first considering the equilibrium conditions at point A. The total force to be transmitted ( $V_b$ ) is assumed to be shared by the stiffener and the web as

$$V_b = \frac{M}{d_b} = d_c w \frac{F_y}{\sqrt{3}} + F_y A_{st} \cos \theta \quad (9.15)$$

where  $A_{st}$  is the total stiffener area required. Solving for this quantity, assuming that all parts have the same yield strength, and introducing the resistance factor—

$$A_{st} = \frac{1}{\phi F_y \cos \theta} \left( \frac{M}{d_b} - \frac{\phi F_y w d_c}{\sqrt{3}} \right) \quad (9.16)$$

Since this element is acting under a compressive load, the ratio of its width to thickness ( $b/t$ ) should be selected so as to avoid the possibility of premature local buckling. (This requirement is discussed in **Chapter 5**.)

For the corner arrangement that has been discussed, groove welds could be used at the junction of the column flanges and the lower flange of the beam. Fillet welds can be used at the other locations to transfer the necessary forces. The resulting fillet weld at the column web to beam flange may be rather large, however, and a groove weld is often used at this location as well. Details of the weld design are given in Example 9.5.

### **Example 9.5**

*Given*

Design the corner connection between a column and a beam, both of which are W410 × 60 sections of G40.21 350W steel ( $F_y = 350$  MPa). Use E49xx electrodes. (The notation of Figure 9.21 will be followed and details of the welding selected will be shown in Figure 9.22.) The beam is to act as a Class 2 section. The behaviour of the column will depend on the magnitude of the axial force.

*Solution*

The connection will be designed to carry the factored moment on the section, which has been determined to be 375 kN · m. The effects of the axial thrust and shear on the

connection web can be neglected. These are small and are of the opposite sign to shears produced by the moment. The factored shear in the column, needed for the design of the weld between the column web and beam flange, is 680 kN.

The web thickness required, using Equation 9.14 is

$$w = \frac{1.9 \times 375 \times 10^6 \text{ N}\cdot\text{mm}}{407 \text{ mm} \times 407 \text{ mm} \times 350 \text{ MPa}} = 12.3 \text{ mm}$$

The web thickness provided by a W410×60 is only 7.7 mm. Therefore, diagonal stiffeners (AD) will be provided and, from Equation 9.16

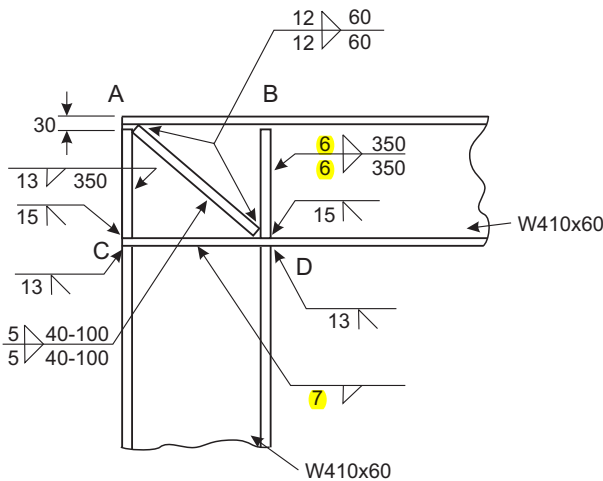
$$A_{st} = \frac{1}{0.90 \times 350 \times 0.707} \left( \frac{375 \times 10^6}{407} - \frac{0.90 \times 350 \times 7.7 \times 407}{\sqrt{3}} \right) = 1578 \text{ mm}^2$$

Provide 789 mm<sup>2</sup> in each of two stiffeners, one on each side of the beam web.

Try 10 mm × 80 mm plates, area = 800 mm<sup>2</sup> each, F<sub>y</sub> = 350 MPa. Although slenderness requirements for such stiffeners are not clear for a joint where the beam is to act as a Class 2 section, it is conservative to use the same slenderness requirement as for the flanges of a Class 2 beam.

$$\text{Allowable } \frac{b}{t} \leq \frac{170}{\sqrt{F_y}} = \frac{170}{\sqrt{350}} = 9.1$$

Actual b/t = 8.0 (Satisfactory)



Welds shown for near side; far side same.

**Figure 9.22 – Design Example – Corner Connection**

Stiffeners at A–C and B–D: Provide 15 mm × 80 mm plates on each side of beam web. This provides the same area (approximately) as the column flanges. A near full-depth stiffener will be used at A–C as shown in Figure 9.22. At B–D, the stiffener can be either full-depth as shown or based upon the weld length required.

Checking, b/t = 80/15 = 5.3 << 9.1 (Satisfactory)

Welds:

Column flanges to beam flange—use complete joint penetration groove welds to develop the full strength of the column flanges.

Column web to beam flange—since it has been assumed that the column flanges carry all the moment in that member, the web will be assumed to carry all the shear (680 kN in this case). For E49xx electrodes, the strength of the weld metal for a 1 mm leg size and for  $\theta = 0^\circ$  (weld axis and force vector parallel) is

$$\begin{aligned} V_r &= 0.67 \phi_w A_w X_u \\ &= 0.67 \times 0.67 \times (1 \text{ mm} \times 0.707) \times 490 \text{ MPa} \\ &= 156 \text{ N/mm} = 0.156 \text{ kN/mm} \end{aligned}$$

The strength of the base metal is

$$\begin{aligned} V_r &= 0.67 \phi_w A_m F_u \\ &= 0.67 \times 0.67 \times 1 \text{ mm} \times 450 \text{ MPa} \\ &= 202 \text{ N/mm} = 0.202 \text{ kN/mm} \end{aligned}$$

The strength of the weld metal governs. The weld length available is  $2 \times 348 = 696$  mm. (The flat portion of the web of a W410  $\times$  60, tabulated as "T" in the CISC Handbook, is 348 mm.)

$$\text{Leg size required} = \frac{680 \text{ kN}}{696 \text{ mm} \times 0.156 \text{ kN/mm/mm}} = 6.3 \text{ mm, say } 7 \text{ mm}$$

Stiffeners—

The forces delivered by the column flanges at C and D by means of the complete joint penetration groove welds between these flanges and the lower beam flange must be transferred into the stiffeners A–C and B–D. This can be accomplished by means of complete joint penetration groove welds at C and D, as shown in Figure 9.22.

The forces now in the stiffeners must be transferred into the beam web, preferably by fillet welds placed along the length of the stiffeners. At either stiffener, this force is (approximately)—

$$F = \frac{375 \times 10^6 \text{ N} \cdot \text{mm}}{407 \text{ mm}} = 921 \times 10^3 \text{ N} = 921 \text{ kN}$$

Weld length available for both stiffeners at A–C  $\approx 348 \times 2 = 696$  mm.

Leg size required (weld strength as calculated for column web to beam flange)

$$= \frac{921 \text{ kN}}{696 \text{ mm} \times 0.156 \text{ kN/mm}} = 8.5 \text{ mm, say } 9 \text{ mm}$$

Stiffener B–D must carry the difference between the flange force (921 kN) and the resistance provided by the beam web. The latter is given by Equation 9.18 (development and explanation to follow) as

$$B_r = \phi_{bi} w_c (t_b + 10t_c) F_{yc}$$

$$B_r = 0.80 \times 7.7 \text{ mm} (12.8 + (10 \times 12.8)) \text{ mm} \times 350 \text{ MPa} = 304 \times 10^3 \text{ N} = 304 \text{ kN}$$

spacing of 100 mm will meet the requirements of S16 Clause 19.1.3(b) and will provide a resistance of

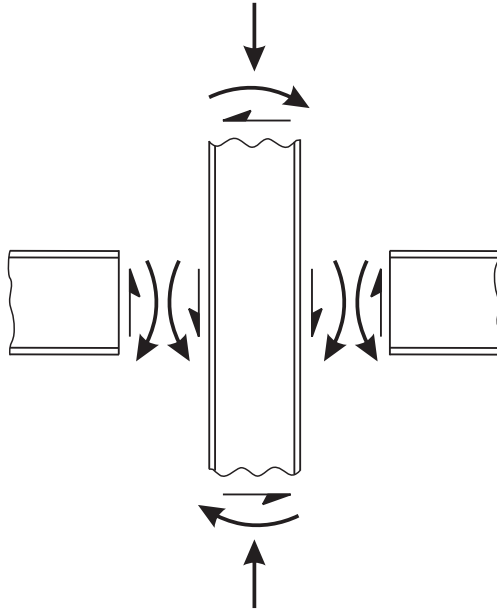
$$V_r = \frac{0.156 \text{ kN} \times 5 \text{ mm} \times 40 \text{ mm}}{100 \text{ mm}} = 0.312 \text{ kN/mm}$$

The load to be transferred is

$$V = \frac{280 \text{ kN}}{980 \text{ mm}} = 0.286 \text{ kN/mm} \quad (\text{Satisfactory})$$

Use an intermittent fillet weld arrangement as shown in Figure 9.22.

The only other type of connection required to carry all three force components that will be discussed is the interior type connection shown in Figure 9.23. An exaggerated view of the deformed connection (Figure 9.24) shows the two possible failure modes; (a) failure of the column web as the beam flange delivers its compressive load, (b) rupture of the groove weld in the stiff region at the beam tension flange.



**Figure 9.23 – Interior Connection**

On the compression side, it will be assumed that the force from the beam flange can be treated as a concentrated load. The design rule for this case was described in Section 5.12. The total factored force from the flange must be less than or equal to the factored web resistance at this point (Equation 5.27(a)). This can be expressed as

$$\frac{M_f}{d_b} \leq B_r \quad \text{or,} \quad \frac{M_f}{d_b} \leq \phi_{bi} w_c (t_b + 10t_c) F_{yc} \quad (9.17)$$

Some adjustments to the notation have been made as compared with Equation 5.27(a). In the latter, the symbol  $N$  is used to represent the length of the bearing plate. In the problem under discussion now, this is simply the thickness of the beam flange,  $t_b$ .

only variable in the analysis, this can be examined first in order to establish the governing case. If the actual web thickness is less than that described by Equation 9.22, a diagonal stiffener or doubler plates would be provided, as was discussed for corner connections.

### Example 9.6

#### Given

The connection of example 9.3 is now to be designed to transfer a bending moment of  $150 \text{ kN} \cdot \text{m}$ , causing compression at the bottom of the connection, in addition to the shear force of  $180 \text{ kN}$  for which the connection was designed in example 9.3.

The required geometric properties of the column sections are:

$$d_c = 318 \text{ mm} ; w_c = 13.1 \text{ mm} ; t_c = 20.6 \text{ mm}$$

The required geometric properties of the beam section are:

$$d_b = 266 \text{ mm} ; t_b = 20.6 \text{ mm}$$

#### Solution

In example 9.3 the connection was designed to transfer the shear force from the beam to the column. This part of the design does not change when the connection is designed to carry both shear and moment. The transfer of moment between the beam and the column is achieved by connecting the beam flanges to the column flange, either by welding the beam flanges directly to the column flange, or by using moment plates. Both options will be illustrated here.

#### Alternative 1 – Welding beam flanges directly to the column

The forces transferred by the beam flanges to the column are determined using Equation 9.13:

$$V_b = \frac{M}{d_b} = \frac{150 \times 10^3 \text{ kN} \cdot \text{mm}}{266 \text{ mm}} = 564 \text{ kN}$$

Because there is only one beam connecting into the column, this force also corresponds to the shear force in the column panel zone. The column web thickness required to resist this shear force is obtained from Equation 9.14.

$$w_c \geq \frac{1.9 M}{d_b d_c F_y} = \frac{1.9 \times 150 \times 10^6 \text{ N} \cdot \text{mm}}{266 \text{ mm} \times 318 \text{ mm} \times 350 \text{ MPa}} = 9.6 \text{ mm}$$

The web thickness provided by a  $W310 \times 129$  is  $13.1 \text{ mm}$ . The panel zone is therefore adequate. The bearing resistance of the column opposite to the beam compression flange is obtained from Equation 9.17.

$$\begin{aligned} B_r &= \phi_{bi} w_c (t_b + 10t_c) F_{yc} = 0.80 \times 13.1 \times (13.0 + (10 \times 20.6)) \times 350 \\ &= 803 \times 10^3 \text{ N} = 803 \text{ kN} \end{aligned}$$

This is larger than the flange force of  $564 \text{ kN}$ . The tension resistance of the flange opposite the beam tension flange can be obtained from Equation 9.18.